NPTEL COURSE - Introduction to Commutative Algebra

Assignment - 4

1. Is $\oplus_{i=1}^{\infty}\mathbb{Z}_{n_i}$, where $n_i \in \mathbb{N}$ a free $\mathbb{Z}$ module ?

2. Let M be a finitely generated A module and $\phi : M \to A^n$ be a surjective homomorphism.

   (a) If $\phi(m_i) = e_i$ for $i = 1, \ldots, n$, then prove that $M = \langle m_1, \ldots, m_n \rangle \oplus \ker \phi$.

   (b) Deduce that $\ker \phi$ is a finitely A-submodule of M.

3. Let $R = k[x^2, xy, xy^2, xy^3, \ldots]$ be polynomial ring over a field k. Prove that $\langle x^2, xy, xy^2, xy^3, \ldots \rangle$ is not finitely generated over k.